

Dielectric (p,q) strings in a throat

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ABSTRACT: We calculate the (p,q) string spectrum in a warped deformed conifold using the dielectric brane method. The spectrum is shown to have the same functional form as in the dual picture of a wrapped D3-brane with electric and magnetic fluxes on its world volume. The agreement is exact in the limit where q is large. We also calculate the dielectric spectrum in the S-dual picture. The spectrum in the S-dual picture has the same form as in the original picture but it is not exactly S-dual invariant due to an interchange of Casimirs of the non-Abelian gauge symmetries. We argue that in order to restore S-duality invariance the non-Abelian brane action should be refined, probably by a better prescription for the non-Abelian trace operation.

KEYWORDS: String theory and cosmic strings, String Duality, D-branes.

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1. Introduction

Recent observations strongly support inflation as the origin of the universe and the structure formation [1]. Despite its impressive success in explaining a host of observational data such as the CMB power spectrum and in solving conceptual puzzles such as the flatness and the horizon problems, the inflationary scenario is still at the phenomenological level. The origin of the inflation and the nature of the inflaton field is not known from a fundamental theory point of view. String theory, on the other hand, is a consistent theory of quantum gravity which is yet to be tested. The energy scale of inflation was probably so high that quantum gravity effects were important, either directly or indirectly. It seems reasonable to expect that string theory, assuming it to be relevant to our universe, will yield many insights into the nature of inflation. If this notion turns out to be true, it would be a unique chance to test the relevance of string theory to our universe.

String theory is a higher dimensional theory. After compactification to four dimensions, many scalar fields or moduli will show up in low energy theory. This has both positive and negative sides. The positive side is that some of these moduli may play the role of the inflation field. In this view, inflation is a natural feature of string theory compactifications. The negative side is that usually many of these moduli will roll quickly. This may violate the slow roll conditions, required for slow roll based inflation. Furthermore, if they are not stabilized and roll quickly they may modify the expansion history of the universe. This can destroy the success of the late time big bang cosmology such as the predictions of the big bang nucleosynthesis for the abundance of the light elements. In this regard the moduli stabilization is a key issue in string cosmology.

There has been interesting progress in moduli stabilization [2–4]. By turning on fluxes it was shown in [3] that, in principle, all complex structures can be stabilized. Some non-perturbative mechanisms are used in [4] to stabilize the Kahler modulus, the

field corresponding to the overall size of the Calabi-Yau (CY) compactification. In the constructions of [3] and [4] there are localized regions inside CY where the geometry of the space-time is warped similar to Randall-Sundrum scenario [5]. The particular model studied is the warped deformed conifold solution of Klebanov-Strassler (KS) [6] glued smoothly to the bulk of the CY manifold. In the language of Randall-Sundrum, the IR region is the bottom of the conifold whereas the UV region is the place where the conifold is smoothly glued to the bulk. The hierarchy of scales between IR and UV is exponentially sensitive to the ratio of fluxes. This way a huge hierarchy of scales can be created with minor tuning of fluxes.

An interesting mechanism to obtain inflation in this set up is to use the brane-antibrane inflationary scenario [7, 8] inside the KS throat [9, 10]. The $\overline{D3}$ -brane is sitting at the IR region of the throat while the D3-brane is moving toward it inside the throat. Due to warp factor, the potential between D3- $\overline{D3}$ becomes flat enough that the slow roll conditions can easily be satisfied. However, the extra contribution from the Kahler modulus of the CY compactification to the inflationary potential conflicts with the slow conditions. It was argued in [9] that a fine-tuning of order one percent is required to keep the inflationary potential flat. Inflation ends when brane and antibrane annihilate each other and this annihilation releases the energy to reheat the universe. After the branes annihilation, fundamental strings (F-strings) and D1-brane (D-strings) are copiously produced. These F and D strings have cosmological size and appear as cosmic strings [11–13]. The range of their tension is compatible with the recent observations [14–16]. The cosmic strings tension in these models is high enough to be detected in near future gravity wave search [17]. Detecting cosmic strings would go a long way in support of brane inflation and string theory.

Since the branes annihilation takes place in the IR region of the KS throat, the cosmic strings are localized in this region, near the tip of the KS throat. The spectrum of cosmic strings is a rich combination of bound states of F and D-strings. It is an interesting question to understand the spectrum of (p, q) strings, the bound states of p F and q D-strings on top of each other at the tip of the throat. This question was studied recently in [18]. In this method, (p, q) strings were viewed as a D3-brane with p units of electric flux and q units of magnetic flux in its world volume, wrapping a two-sphere inside the three-sphere at the tip of the KS throat. The (p, q) spectrum was shown to be in agreement with the flat space formula. Furthermore, in the case $q = 0$ it reproduces the result of [25] for tension spectrum of p F-strings in $SU(M)$ gauge theory.

One may try to calculate the (p, q) spectrum directly, using the dielectric brane method [19]. In this picture, q D-strings on top of each other blow up into a fuzzy D3-brane, while the fundamental strings are viewed as p units of electric flux on D-strings world volume. The extra two coordinates on the world volume of the fuzzy D3-brane corresponds to the positions of q D-strings in the orthogonal directions. These orthogonal dimensions span a two-sphere inside the three-sphere at the tip of the throat. In a sense, the dielectric brane method of calculating the (p, q) spectrum is the reverse direction of calculating the (p, q) spectrum using the wrapped D3-brane method in [18]. In general, one expect that for large q these two methods give the same result for the spectrum [19, 20].

The paper is organized as follows. In next section we briefly review the KS throat. In section 3 the (p, q) spectrum is calculated using the dielectric brane method. In section 4 this calculation is carried on for the S-dual background of KS throat. The Conclusions are given in section 5.

2. The warped deformed conifold

In this section we briefly review the warped deformed conifold. A cone is defined by the following equation in \mathbf{C}^4 [21]

$$\sum_{i=1}^4 w_i^2 = 0. \tag{2.1}$$

This describes a smooth surface apart from the point $w_i = 0$. The base of the cone is given by the intersection of eq. (2.1) with a sphere of radius r in R^8 ,

$$\sum_i |w_i|^2 = r^2$$

We are interested in Ricci-flat metrics on the cone which in turn imply that the base of the conifold is a Sasaki-Einstein manifold. The simplest five dimensional Sasaki-Einstein manifold for $N = 1$ supersymmetry is $T^{1,1}$ and it is the only manifold for which the deformation is explicitly known [6].

The metric on the conifold with base $T^{1,1}$ is

$$\begin{aligned} ds_6^2 &= dr^2 + r^2 ds_{T^{1,1}}^2 \\ ds_{T^{1,1}}^2 &= \frac{1}{9} (d\psi + \sum_{i=1}^2 \cos \theta_i d\phi_i)^2 + \frac{1}{6} \sum_{i=1}^2 (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2). \end{aligned} \tag{2.2}$$

It can be shown that $T^{1,1}$ has topology of $S^2 \times S^3$ with S^2 fibered over S^3 . If φ_1 and φ_2 are the two Euler angles of the two S^3 's, respectively, then their difference corresponds to $U(1)$ while $\psi = \varphi_1 + \varphi_2$. Since $2\pi \geq \varphi_i \geq 0$, the range of ψ is $[0, 4\pi]$.

The Klebanov-Strassler throat that we are interested in is actually a warped deformed conifold. This warped deformed conifold emerges in the presence of fluxes. The R-R flux F_3 wraps the S^3 at the bottom of the deformed conifold, while NS-NS flux H_3 wraps the dual 3-cycle B that generates the warped throat

$$\frac{1}{4\pi^2 \alpha'} \int_B H_3 = -K, \quad \frac{1}{4\pi^2 \alpha'} \int_{S^3} F_3 = M. \tag{2.3}$$

The metric of the deformed conifold is studied in [22–24]. At the tip of the deformed conifold one S^2 shrinks to zero size and the internal geometry reduces to a round S^3 . At the tip the metric is given by

$$ds^2 \sim h^2 \eta_{\mu\nu} dx^\mu dx^\nu + b g_s M \alpha' (d\psi^2 + \sin^2 \psi d\Omega_2^2) \tag{2.4}$$

where h is the warp factor at the bottom of the throat

$$h = \epsilon^{2/3} 2^{-1/6} a_0^{-1/4} (g_s M \alpha')^{-1/2} \tag{2.5}$$

where $a_0 \sim .72$ and $b = 2^{2/3} 3^{-1/3} I(0)^{1/2} \sim 0.93$ are numerical constants in the KS solution and $\epsilon^{1/3}$ is the deformation radius. Here ψ is the usual azimuthal coordinate in a S^3 ranging from 0 to π .

There are M units of RR 3-forms F_3 on the non-vanishing S^3 cycle at the tip of the throat. Its associated two form is given by

$$C^{(2)} = M \alpha' \left(\psi - \frac{\sin(2\psi)}{2} \right) \sin \theta d\theta d\phi. \tag{2.6}$$

In KS solution $C_0 = 0$ and at the bottom of the throat $B_{ab} = 0$. Furthermore, on the gauge theory side M corresponds to $SU(M)$ gauge theory living at the tip of the conifold.

3. The dielectric (p,q) strings

One may consider to obtain the bound states of (p, q) strings directly [26], using the non-commutative dielectric brane method prescribed by Myers [19]. Upon expansion, the results of [26] were in agreements with the results of [18] up to order $1/M^2$. It is an interesting exercise to see whether there is also an exact agreement between the non-commutative dielectric brane method and the dual method of wrapped D3-brane used in [18].

When N p-branes are located on top of each other, the ground state of open strings attached between them becomes massless and the $U(1)^N$ symmetry associated with N individual branes is enhanced to $U(N)$. The gauge field vector A_a becomes non-Abelian and

$$A_a = A_a^{(n)} T_n \quad , \quad F_{ab} = \partial_a A_b - \partial_b A_a + i[A_a, A_b], \tag{3.1}$$

where T_n are N^2 Hermitian generators with $Tr(T_n T_m) = N \delta_{nm}$. The orthogonal displacements of branes, Φ^i , are now matrix valued and transform in the adjoint of $U(N)$ with

$$D_a \Phi^i = \partial_a \Phi^i + i[A_a, \Phi^i]. \tag{3.2}$$

The non-Abelian action of N coincident p-branes is given by $S = S_{DBI} + S_{CS}$, where [19]

$$S_{DBI} = -\mu_p \int d^{p+1} \sigma \text{STr} \left(e^{-\phi} \sqrt{\det(Q^i_j)} \sqrt{-\det(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij} E_{jb}] + \lambda F_{ab})} \right) \tag{3.3}$$

and

$$S_{CS} = \mu_p \int \text{STr} \left(P \left[e^{i\lambda \Phi^i \Phi^i} \left(\sum C^{(n)} e^B \right) \right] e^{\lambda F} \right). \tag{3.4}$$

In these expressions, $\lambda = 2\pi\alpha'$, μ_p is the p-brane charge, $E_{MN} = g_{MN} + B_{MN}$, $Q^i_j \equiv \delta^i_j + i\lambda [\Phi^i, \Phi^k] E_{kj}$ and Φ^i is the branes location along the orthogonal direction x^i . In the

conventions of [19], $x^i = \lambda \Phi^i$, so that Φ^i has dimensions of mass. The indices a, b, \dots are along the brane world-volume directions, while i, j, \dots represents the directions orthogonal to the branes. The operation STr corresponds to the trace of symmetrised pairing of the non-Abelian fields F_{ab} and Φ^i [27]. All induced quantities are the pull-backs of space-time tensors on the brane.

In S_{CS} , the operator i_Φ denotes the interior product acting on an n -form $C^{(n)} = \frac{1}{n!} C_{M_1 \dots M_n}^{(n)} dx^{M_1} \dots dx^{M_n}$ and

$$i_\Phi i_\Phi C^{(n)} = \frac{1}{2(n-2)!} [\Phi^i, \Phi^j] C_{jiM_3 \dots M_n}^{(n)} dx^{M_3} \dots dx^{M_n}. \quad (3.5)$$

To calculate the (p, q) spectrum, one can turn on p units of $U(1)$ electric field on the world volume of q coincident D1-branes. We take the D1-branes to be extended in the (t, x^μ) direction and the only relevant component of F_{ab} is $F_{0\mu}$. Furthermore, at the tip of KS throat $B_{MN} = 0$ and $E_{MN} = g_{MN}$. The only non-zero term in S_{CS} is the term coming from the operation of i_Φ on $C^{(2)}$.

With these assumptions the action of q D1-branes with electric fields on their world volumes is

$$S = \int dt dx^\mu \left[-\frac{\mu_1}{g_s} \sqrt{h^2 - \lambda^2 F_{0\mu}^2} \text{STr} \sqrt{\det(Q_j^i)} + \frac{i\lambda}{2} F_{0\mu} \text{STr} \left([\Phi^i, \Phi^j] C_{ji}^{(2)} \right) \right]. \quad (3.6)$$

To simplify the algebra it is useful to write the above action in the form

$$S = \int dt dx^\mu \left[-\Delta \sqrt{h^4 - \lambda^2 F_{\mu 0}^2} + \Omega F_{0\mu} \right], \quad (3.7)$$

with

$$\Delta \equiv -\frac{\mu_1}{g_s} \text{STr} \sqrt{\det(Q_j^i)} \quad , \quad \Omega \equiv \frac{i\lambda}{2} \text{STr} \left([\Phi^i, \Phi^j] C_{ji}^{(2)} \right) \quad (3.8)$$

The conjugate momentum associate with the electric field is

$$D = \frac{\delta \mathcal{L}}{\delta F_{0\mu}} = \frac{\Delta \lambda^2 F_{0\mu}}{\sqrt{h^4 - \lambda^2 F_{\mu 0}^2}} + \Omega. \quad (3.9)$$

Using this expression to eliminate $F_{0\mu}$, the Hamiltonian is [18]

$$\begin{aligned} \mathcal{H} &= DF_{\mu 0} - \mathcal{L} \\ &= \frac{h^2}{\lambda} \sqrt{\Delta^2 \lambda^2 + (D - \Omega)^2} \end{aligned} \quad (3.10)$$

Our goal is to calculate Δ and Ω for the model at hand and plug them in the above expression for the Hamiltonian. Furthermore, since the F-strings in (p, q) strings are associated with the electric fields this means $D = p$.

Motivated by the dual wrapped D3-brane picture, we expect that q D1-branes blow up into a fuzzy D3-brane. This fuzzy D3-brane wraps a two sphere inside the S^3 at the tip of the deformed conifold. The transverse coordinates x^i span the surface of this two

sphere, so Φ^i belongs to a q -dimensional representation of the $SU(2)$ algebra. We take the following ansatz for the transverse coordinates [28]

$$\Phi^i = \hat{R} \alpha^i \quad (3.11)$$

where the constant \hat{R} with dimension mass will be determined later and α^i are the $q \times q$ dimensional matrix representation of $SU(2)$ algebra

$$[\alpha^i, \alpha^j] = 2i\epsilon_{ijk}\alpha^k \quad (3.12)$$

It is important to realize that the metric on the space spanned by the transverse coordinates x^i is flat and $g_{kj} = \delta_{kj}$. The radius of this S^2 is basically the same as the radius of S^2 given in eq. (2.4), i.e.

$$R^2 = \sum_{i=1}^3 (x^i)^2 = b g_s M \alpha' \sin^2 \psi \quad (3.13)$$

On the other hand

$$R^2 = \frac{\lambda^2}{q} \sum_{i=1}^3 \text{Tr}[(\Phi^i)^2] = \lambda^2 C \hat{R}^2 \quad (3.14)$$

which can be used to fix the constant \hat{R}

$$\hat{R} = \frac{R}{\lambda \sqrt{C}}. \quad (3.15)$$

Here C is the Casimir of the representation. For q -dimensional irreducible representation of $SU(2)$ which will be used in our calculation, $C = C_q = q^2 - 1$.

With these assumptions, we obtain

$$Q_j^i = \delta_j^i - 2\epsilon_{ijk}\lambda\hat{R}\Phi^k \quad (3.16)$$

and

$$\det(Q_j^i) = 1 + 4\lambda^2 \hat{R}^2 \sum_{i=1}^3 (\Phi^i)^2 + \dots \quad (3.17)$$

where the extra terms in eq. (3.17) are terms which are not symmetric under exchange of Φ^i and do not contribute in $S\text{Tr}$ operation. One finds

$$\begin{aligned} \Delta &= \frac{\mu_1}{g_s} \text{STr} \sqrt{\det(Q_j^i)} = \frac{\mu_1}{g_s} \left(1 + 4\lambda^2 \hat{R}^2 \sum_{i=1}^3 (\Phi^i)^2 \right)^{1/2} \\ &= \frac{q}{\lambda g_s} \left(1 + \left(\frac{b g_s M}{\pi \sqrt{C_q}} \right)^2 \sin^4 \psi \right)^{1/2} \end{aligned} \quad (3.18)$$

where in the last line the relation $\mu = \lambda^{-1}$ is used and R is given by eq. (3.13).

To calculate Ω , one needs to transform $C^{(2)}$ in eq. (2.6) from $C_{\theta\phi}^{(2)}$ components into the Cartesian coordinates $C_{ij}^{(2)}$. For example, to calculate $C_{12}^{(2)}$, we have

$$\begin{aligned} C_{12} &= \left(\frac{\partial\theta}{\partial x^1} \frac{\partial\phi}{\partial x^2} + \frac{\partial\phi}{\partial x^1} \frac{\partial\theta}{\partial x^2} \right) C_{\theta\phi} \\ &= \frac{x_3 C_{\theta\phi}}{R^2 \sqrt{(x_1^2 + x_2^2)}} \end{aligned} \quad (3.19)$$

Performing this coordinate transformation one obtains

$$\left(C_{12}^{(2)}, C_{23}^{(2)}, C_{31}^{(2)} \right) = \frac{\lambda}{R^3} M \alpha' (\psi - \frac{1}{2} \sin 2\psi) (\Phi_3, \Phi_1, \Phi_2) \quad (3.20)$$

and

$$\epsilon_{ijk} \Phi^k C_{ij}^{(2)} = \frac{2}{\lambda R} (\psi - \frac{1}{2} \sin 2\psi) I_q \quad (3.21)$$

where I_q is the q -dimensional unit matrix. This gives

$$\begin{aligned} \Omega &= \frac{i\lambda}{2} \text{STr} \left([\Phi^i, \Phi^j] C_{ji}^{(2)} \right) \\ &= \frac{qM}{\pi \sqrt{C_q}} (\psi - \frac{1}{2} \sin 2\psi) \end{aligned} \quad (3.22)$$

Having calculated Δ and Ω and setting $D = p$, we can use eq. (3.10) to calculate the Hamiltonian of the system

$$\mathcal{H} = \frac{h^2}{\lambda} \left(\frac{q^2}{g_s^2} + \frac{1}{1 - 1/q^2} \frac{b^2 M^2}{\pi^2} \sin^4 \psi + \left[p - \frac{1}{\sqrt{1 - 1/q^2}} \frac{M}{\pi} \left(\psi - \frac{\sin 2\psi}{2} \right) \right]^2 \right)^{1/2} \quad (3.23)$$

The stable solutions, obtained by minimizing the Hamiltonian, are

$$\left(\psi + \frac{b^2 - 1}{2} \sin 2\psi \right) = \sqrt{1 - 1/q^2} \frac{p\pi}{M} \quad (3.24)$$

and the energy of the (p, q) system, $T_{(p,q)}$, is

$$T_{(p,q)} = \mathcal{H}_{min} = \frac{h^2}{\lambda} \left(\frac{q^2}{g_s^2} + \frac{1}{1 - 1/q^2} \frac{b^2 M^2}{\pi^2} \sin^2 \psi (1 + (b^2 - 1) \cos^2 \psi) \right)^{1/2} \quad (3.25)$$

where ψ is given by the minimization equation (3.24).

In the limit that $b^2 - 1 \rightarrow 0$, the stable solutions are given by

$$\psi = \sqrt{1 - 1/q^2} \frac{p\pi}{M} \quad (3.26)$$

and the energy of the system is

$$T_{(p,q)} = \frac{h^2}{\lambda} \sqrt{\frac{q^2}{g_s^2} + b^2 \left(\frac{M}{\pi \sqrt{1 - 1/q^2}} \right)^2 \sin^2 \left(\frac{\pi \sqrt{1 - 1/q^2}}{M} p \right)} \quad (3.27)$$

Now we can compare the above energy spectrum of non-commutative (p, q) strings with the result of the dual method of wrapped D3-brane in [18]. In this method the bound states of (p, q) strings are interpreted as a D3-brane with world volume electric and magnetic fluxes wrapping some two cycles inside the three sphere at the bottom of the KS throat. The flux configurations on a D3 wrapping a 2-cycle that can induce the charge of p F-strings and q D-strings are

$$\tilde{F}^{0\mu} = -\frac{p}{4\pi} \qquad F_{\theta\phi} = \frac{q}{2} \qquad (3.28)$$

where \tilde{F} is the conjugate field associated with the electric field given by $\tilde{\mathcal{F}}^{\mu\nu} = -\delta\mathcal{L}/\delta F_{\mu\nu}$.

The D3-brane action is

$$S_{D3} = -\frac{\mu_3}{g_s} \int d^4\xi \sqrt{-|g_{ab} + \mathcal{F}_{ab}|} + \mu_3 \int C^{(2)} \wedge \mathcal{F}, \qquad (3.29)$$

where $\mathcal{F}_{ab} = B_{ab} + \lambda F_{ab}$.

After integrating out fluxes and the angular components of the metric, the Hamiltonian takes the form

$$\mathcal{H} = \frac{h^2}{\lambda} \sqrt{\frac{q^2}{g_s^2} + \frac{b^2 M^2}{\pi^2} \sin^4 \psi + \left[p - \frac{M}{\pi} \left(\psi - \frac{\sin 2\psi}{2} \right) \right]^2}. \qquad (3.30)$$

with the energy

$$E_{(p,q)} = \frac{h^2}{\lambda} \sqrt{\frac{q^2}{g_s^2} + b^2 \left(\frac{M}{\pi} \right)^2 \sin^2 \left(\frac{\pi}{M} p \right)} \qquad (3.31)$$

Comparing the energy in eq. (3.31) with the energy obtained from non-commutative method in eq. (3.27) we see the agreement is very close, except with the extra coefficient $\sqrt{1 - 1/q^2}$. In the limit where $q \rightarrow \infty$ these two results agree exactly. The interpretation is that in this limit the non-commutative behavior of Φ^i becomes unimportant and one reaches the Abelian limit. This also happens in the examples studied in [19, 20]. For example when N D0-branes are on top of each other in a constant four field strength F_{tijk} , they will blow up into a fuzzy D2-brane. Up to the extra factor $\sqrt{1 - 1/N^2}$, the energy of the dielectric system matches with the energy calculated from the dual picture of a D2-brane with magnetic fluxes wrapping a two cycle.

In the limit that $M \rightarrow \infty$, our formula eq. (3.27) reproduces the spectrum of (p, q) strings in flat space-time. This is expected, since in this limit the size of S^3 is very large and the metric (2.4) approximates the metric of a flat space-time. If $p = 0$, we obtain the tension of q D-strings with

$$T_{D1} = \frac{h^2}{2\pi\alpha'} \frac{q}{g_s}. \qquad (3.32)$$

To obtain the energy of p fundamental strings one can not simply set $q = 0$, since in the non-commutative method the fundamental strings were viewed as electric flux on the world volume of q D-strings. To obtain the energy of fundamental strings one can use the S-dual picture, which is the subject of next section.

4. The dielectric (p,q) strings in the S-dual picture

The **IIB** string theory enjoys an exact symmetry, called the S-duality. Under S-duality transformation the strongly coupled string theory is mapped into the weakly coupled theory. To be more specific let us denote the quantities in the S-dual picture by the superscript S. Under the S-duality transformation $g_s \rightarrow g_s^S = 1/g_s$. Furthermore, the (p, q) system is mapped into $(p^S, q^S) = (-q, p)$, while $B^{(2)S} = -C^{(2)}$, $C^{(2)S} = B^{(2)}$ and $C^{(4)}$ remains invariant.

To obtain the energy of p fundamental strings in the KS solution, we can perform an S-duality transformation. The fundamental strings in original KS background are now p coincident D-strings in the S-dual picture. Furthermore, in the S-dual picture with p coincident D-strings we can turn on $-q$ units of electric field on their world volume to obtain the spectrum of $(-q, p)$ strings. Our dielectric brane method used in the previous section can now be applied again to obtain the energy spectrum. Since the S-duality is exact these two energy spectra should be equal to each other, i.e.

$$T_{(p,q)} = T_{(-q,p)}^S \tag{4.1}$$

where $T_{(p,q)}$ is the energy of (p, q) strings calculated in original KS background given in eq. (3.27) while $T_{(-q,p)}^S$ is the energy of $(p^S, q^S) = (-q, p)$ strings in the S-dual picture.

In the S-dual picture, $C^{(2)S} = B^{(2)} = 0$ at the tip of the throat, while the Kalb-Ramond field $B^{(2)S} = -C^{(2)}$ is not zero and is given by eq. (2.6). Also $g_{MN}^S = g_s^{-1/2} g_{MN}$, so $h \rightarrow g_s^{-1/2} h$ and $R \rightarrow g_s^{-1/2} R$. There is no contribution from S_{CS} in the action, but there are new contributions in S_{DBI} due to extra terms in Q_j^{iS} . We have

$$\begin{aligned} Q_j^{iS} &= \delta_j^i + i\lambda [\Phi^i, \Phi^j] g_{kj} - i\lambda [\Phi^i, \Phi^j] C_{kj}^{(2)} \\ &\equiv Q_j^i + \delta Q_j^i \end{aligned} \tag{4.2}$$

where the first term, Q_j^i , is the same as before given in eq. (3.16), while the last term is the change in Q_j^i in the S-dual picture. One can show that

$$(\delta Q_1^1, \delta Q_2^2, \delta Q_3^3) = -\frac{2\lambda^2 \hat{R}}{R^3} (\psi - \frac{1}{2} \sin 2\psi) (\Phi_2^2 + \Phi_3^2, \Phi_1^2 + \Phi_3^2, \Phi_1^2 + \Phi_2^2) \tag{4.3}$$

and

$$\delta Q_j^i = \frac{2\lambda^2 \hat{R}}{R^3} (\psi - \frac{1}{2} \sin 2\psi) \Phi_j \Phi_i. \quad i \neq j \tag{4.4}$$

This gives

$$\text{STr} \sqrt{\det(Q_j^i)} = p \sqrt{\frac{4R^4}{\lambda^2 C_p} + \left(1 - \frac{M}{\pi \sqrt{C_p}} (\psi - \frac{1}{2} \sin 2\psi)\right)^2} \tag{4.5}$$

where $C_p = p^2 - 1$ is the Casimir associated with the p -dimensional irreducible representation of $SU(2)$.

Our action in the S-dual picture now takes the desired form eq. (3.7) with Δ given by equations (3.8) and (4.5), while $\Omega = 0$ since $C^{(2)S} = 0$. Furthermore, we have $-q$ units of electric fluxes so in going to Hamiltonian we set $D = -q$. The Hamiltonian of the system is

$$\mathcal{H}^S = \frac{h^2}{\lambda} \left(\frac{q^2}{g_s^2} + \frac{1}{1-1/p^2} \frac{b^2 M^2}{\pi^2} \sin^4 \psi + \left[p - \frac{1}{\sqrt{1-1/p^2}} \frac{M}{\pi} \left(\psi - \frac{\sin 2\psi}{2} \right) \right]^2 \right)^{1/2} \quad (4.6)$$

This Hamiltonian has exactly the same form as the Hamiltonian (3.23) in the original KS background, except C_q is replaced by C_p , i.e. $1/\sqrt{1-1/q^2} \rightarrow 1/\sqrt{1-1/p^2}$. Following the same steps which lead to eq. (3.27), for the energy of $(-q, p)$ system in the S-dual picture in the limit $b^2 - 1 \rightarrow 0$ we obtain

$$T_{(q,p)}^S = \frac{h^2}{\lambda} \sqrt{\frac{q^2}{g_s^2} + b^2 \left(\frac{M}{\pi \sqrt{1-1/p^2}} \right)^2 \sin^2 \left(\frac{\pi \sqrt{1-1/p^2}}{M} p \right)}. \quad (4.7)$$

Curiously enough, despite the interesting functional similarity between eq. (3.27) and eq. (4.7), $T_{(p,q)}$ is not exactly equal to $T_{(q,p)}^S$, whereas under S-duality they must be identical. The origin of this discrepancy is the fact that under S-duality the dimension of the irreducible representation of $SU(2)$ has changed from q into p . This has the effect of changing C_q into C_p . The spectrum is S-dual invariant only if C_q and C_p is replaced by q^2 and p^2 , respectively.

One may wonder if S-duality invariance is in fact satisfied in the dual wrapped D3-brane method. Following the calculations of [18] in the S-dual picture, one can show that indeed the energy spectrum is S-dual invariant, i.e. $E^S(p^S, q^S) = E(p, q)$ where $(p^S, q^S) = (-q, p)$ and $E(p, q)$ is given in eq. (3.31). This is expected, since the D3-brane is known to be manifestly S-dual invariant [29].

We are now able to calculate the energy of p fundamental strings in KS solution. It is given by setting $q = 0$ in eq. (4.7), which agrees with the result of [25], again except with the extra factor $1/\sqrt{1-1/p^2}$. The stable solutions of p fundamental strings are given by

$$\psi = \sqrt{1-1/p^2} \frac{p \pi}{M}. \quad (4.8)$$

The interesting new feature is that due to the extra factor $\sqrt{1-1/p^2}$ the tension of p coincident fundamental strings does not vanish when $p = M$. In the analysis of [25], when $p = M$ the energy vanishes. From the gauge theory point of view it was argued that fundamental strings corresponds to QCD flux tubes connecting p quarks and p anti-quarks of $SU(M)$ gauge theory at the tip of the conifold. When $p = M$, these quarks and anti-quarks confine to create the baryon and anti-baryon and the fluxes disappear. If one to take this argument seriously in the light of AdS/CFT correspondence, then this requires that the square root in eq. (4.8) should be replaced by unity which is equivalent to replacing C_p by p^2 . This is also supported by the S-duality invariance demand as described above.

5. Discussion

Using the dielectric brane prescription of Myers the (p, q) strings spectrum is calculated in KS throat. With q D-strings on top of each other, they blow up into a fuzzy two sphere. This represents a bound state of a D3-brane with q D1-branes smeared on it. The F-strings are incorporated in the model by turning on electric fluxes on the world volume of D-strings. The result for energy spectrum is in interesting agreement with the dual picture, where the (p, q) string is interpreted as a wrapped D3-brane with p units of electric flux and q units of magnetic flux in its world volume. The agreement is exact when $q \rightarrow \infty$, as expected.

Since S-duality is an exact symmetry of **IIB** string theory, one expects that the energy spectrum calculated in the S-dual picture is the same as in the original picture. Although the energy spectrum has exactly the same functional form in both pictures, however there are interesting parametric discrepancies. This is due to an interchange of the Casimir in two pictures. In the original picture Φ^i represent a q -dimensional irreducible representation of $SU(2)$ algebra, whereas in the S-dual picture they belong to a p -dimensional irreducible representation of $SU(2)$ algebra. The agreement is exact only when both $p, q \rightarrow \infty$.

This raises the question whether the dielectric brane prescription of [19] is invariant under S-duality in **IIB** string theory. Obviously, to recover the S-duality in this calculations one needs to “effectively” replace C_p and C_q by p^2 and q^2 , respectively. One may try to add some new terms in equations (3.3) and (3.4) to restore the S-duality. Although this is a possibility, but it should be performed in way not to spoil the T-duality invariance of the original prescription. The most probable cause of S-duality violation may be the symmetric trace prescription used in (3.3) and (3.4). It is known that the symmetric trace prescription does not agree with the full effective string action [30, 31] and corrections of order F^6 and higher in the world-volume field strength should be added to the action. It is interesting to know whether a better prescription for the trace operation in the non-Abelian action can resolve the problem. If so, the benefits are two folds: not only the S-duality is restored, but also the dielectric calculation matches exactly with the result of dual method for any value of q . This happens in (p, q) string case here and also for the case in [19] where N D0-branes blow up into a D2 brane.

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